Find Z-inverse Transform

$X(z) = \quad 1 \quad 1$

Differentiation in freq. DTFT.

If $\mathcal{F}[x(n)] = X(e^{j\omega})$

$$\mathcal{F}[n\,x(n)] = j\frac{d}{d\omega}(X(e^{j\omega}))$$

$$X(e^{j\omega}) = \mathcal{F}[x(n)] = \sum_{n=-\infty}^{\infty} x(n)\,e^{-j\omega n} \quad \frac{d}{d\omega}$$

Differentiation in both side

$$\frac{dX(e^{j\omega})}{d\omega} = \sum_{n=-\infty}^{\infty} (-jn)\,x(n)\,e^{-j\omega n}$$

$$= -j\sum_{n=-\infty}^{\infty} (n)\,x(n)\,e^{-j\omega n}$$

$$j\frac{dX(e^{j\omega})}{d\omega} = \sum_{n=-\infty}^{\infty} n\,x(n)\,e^{-j\omega n}$$

$$= \mathcal{F}[n\,x(n)]$$

$$\boxed{\mathcal{F}[n\,x(n)] = j\frac{dX}{d\omega}(e^{j\omega})}$$

$$n\sum_{n=-\infty}^{\infty} x(n)\,e^{-j\omega n}$$

$$n(X(e^{j\omega}))$$

$$\mathcal{F}[n \cdot x(n)]$$

⇒ Convolution

If $\mathcal{F}\,x_1(n) \rightarrow X_1(e^{j\omega})$

$$\mathcal{F}[x_2(n)] = X_2(e^{j\omega})$$

$$\mathcal{F}[x_1(n) * x_2(n)] = X_1(e^{j\omega}) \cdot X_2(e^{j\omega})$$

Multiplication

time.

$$\mathcal{F}(x_1(n) * x_2(n)) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)\, e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} x(k) \underbrace{\sum_{n=-\infty}^{\infty} h(n-k)\, e^{-j\omega n}}$$

Put $n-k = P$, then $n = P+k$. $\quad n = (P+k)$

$e^{-j\omega(P+k)}$

$$\mathcal{F}[x_1(n) * x_2(n)] = \sum_{k=-\infty}^{\infty} x(k) \underbrace{\sum_{P=-\infty}^{\infty} h(P)\, e^{-j\omega P}}\, e^{-j\omega k}$$

$$= \sum_{k=-\infty}^{\infty} x(k)\, H(e^{j\omega})\, e^{-j\omega k}$$

$$= H(e^{j\omega}) \sum_{k=-\infty}^{\infty} x(k)\, e^{-j\omega k}$$

$$= \boxed{H(e^{j\omega}) \cdot X(e^{j\omega})}$$

Proved.

$$= \sum_{n=-\infty}^{\infty} (x_1(n) * x_2(n))\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x_1(k) \cdot x_2(n-k)\, e^{-j\omega n}$$

## Fourier Transform of Periodic Signal

As we know that DTFS is used for periodic signal → F.T is directly achieved through Fourier series. $Lt_{N \to \infty}$

Now the time period will be taken as $2\pi$ which will be repeating.

$\omega_0 = \frac{2\pi}{N}$

DTFS, $x(n) = \sum_{k=N} X_k e^{jk\omega_0 n}$

Take Fourier Transform on both sides

$$\mathcal{F}[x(n)] = \mathcal{F}\left[\sum_{k=N} X_k e^{j\omega_0 kn}\right]$$

$\mathcal{F}.e^{j\omega_0 t} = \delta(\omega-\omega_0)$

$$X(e^{j\omega}) = \sum_{k=N} X_k \, \mathcal{F}\, e^{(j\omega_0 kn)}$$

$\mathcal{F}(1).e^{j\omega_0 kn}$ frequency shifting

We know that $\mathcal{F}(e^{j\omega_0 kn}) = 2\pi\delta(\omega - k\omega_0)$

$0 \le \omega \le 2\pi$

$$X(e^{j\omega}) = \sum_{k=N} X_k \, 2\pi\delta(\omega - k\omega_0) \quad 0 < \omega <$$

$$\boxed{X(e^{j\omega}) = 2\pi \sum_{k=N} X_k \, 2\pi\delta(\omega - k\omega_0)}$$

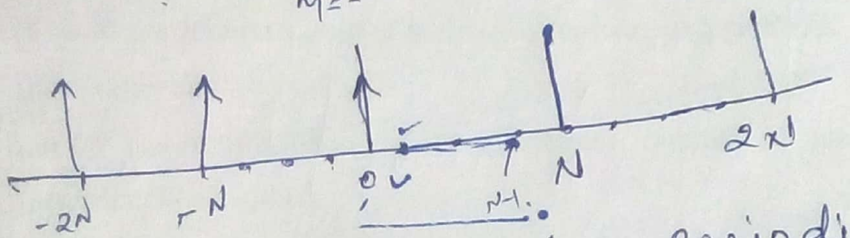
Impulse

DTFT is periodic with period $\frac{2\pi}{N}$, it follows that $X(e^{j\omega})$ consists a set of impulses of strength $2\pi X_k$, $k = 0, 1, 2 \ldots$ separated at intervals of $N\omega_0 =$

Thus the Fourier Transform of a periodic signal is simply an impulse train with impulses located at $\omega = k\omega_0$ — all with a strength of $2\pi X_k$ and each other impulses are separated from each other by $\omega_0$.

Q1. Find the Fourier Transform of the discrete-time impulse train.

$$x(n) = \sum_{m=-\infty}^{\infty} \delta(n - mN).$$



Sol: The signal is periodic with period $N$ and frequency $\omega_0 = \frac{2\pi}{N}$

$$x(n) = \begin{cases} 1, & n = 0 \\ 0, & 1 \le n \le N-1 \end{cases}$$

$$= \delta(n)$$

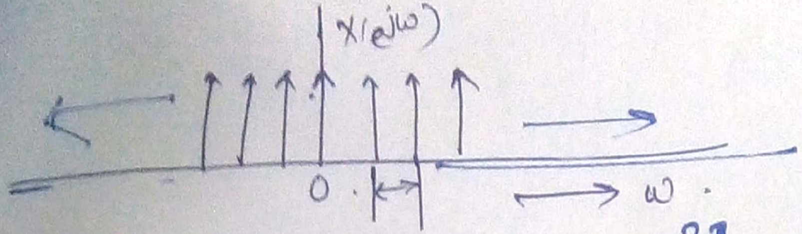
$$X_k = \frac{1}{N} \sum_{0}^{N-1} x(n) e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=0}^{N-1} \delta(n) e^{-jk\left(\frac{2\pi}{N}\right) n}$$

Spectrum DTFS

$$X_k = \frac{1}{N} e^{-jk\left(\frac{2\pi}{N}\right)n} \Big|_{n=0} = \frac{1}{N}$$

$$\text{DTFT} = X(e^{j\omega}) = 2\pi \sum_{k=-\infty}^{\infty} X_k \, \delta(\omega - k\omega_0)$$

$$= 2\pi \sum \frac{1}{N} \delta(\omega - k\omega_0) = \frac{2\pi}{N} \sum^{\infty}$$




$\omega_0 = \frac{2\pi}{N}$

DTFT of a discrete time impulse train

Q. Find the fourier Transform of periodic signal

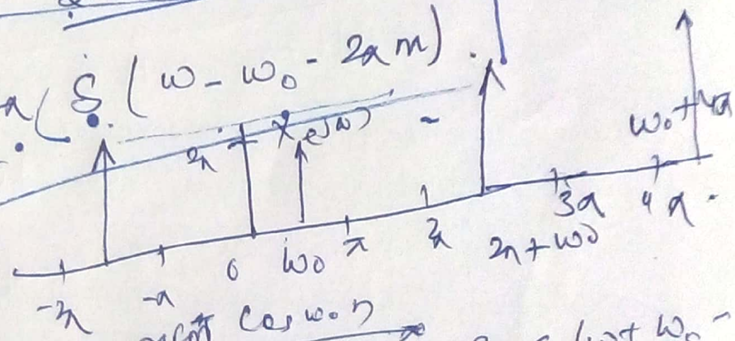
$x(n) = \cos(\omega_0 n)$

$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$

$\omega_0 = \frac{2\pi}{5}$

$x(n) = \cos\omega_0 n = \frac{1}{2}\left(e^{j\omega_0 n} + e^{-j\omega_0 n}\right)$

$e^{j\omega_0 n} \overset{\text{DTFT}}{\longleftrightarrow} \sum_{m=-\infty}^{\infty} 2\pi\,\delta(\omega - \omega_0 - 2\pi m)$




DTFT $\cos\omega_0 n$

$= \frac{1}{2}\sum_{m=-\infty}^{\infty} 2\pi\,\delta(\omega - \omega_0 - 2\pi m) + \frac{1}{2}\sum_{m=-\infty}^{\infty} 2\pi\,\delta(\omega + \omega_0 - 2\pi m)$

$\omega_0 = \frac{2\pi}{5}$

$= \frac{1}{2}\sum_{m=-\infty}^{\infty} 2\pi\,\delta\left(\omega - \frac{2\pi}{5} - 2\pi m\right) + \frac{1}{2}\sum_{m=-\infty}^{\infty} 2\pi\,\delta\left(\omega + \frac{2\pi}{5} - 2\pi m\right)$

freq. $e^{+\omega_0} \rightleftharpoons \delta(\omega - \omega_0)$

$e^{-\omega_0} \rightleftharpoons \delta(\omega + \omega_0)$

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